the three closed-form solutions for the heading or yaw angle. It is noted that the solution just discussed agrees better with the numerical data than does the "spherical earth" solution. At large values of $W/C_DA\rho$, the preceding solution is better by approximately 50%, whereas at small values of $W/C_DA\rho$, the difference between the foregoing solution and the numerical data is extremely small. In view of the solution contained herein, it is apparent that a better closed-form solution for the heading angle may be obtained by omitting the "lateral centrifugal force" term and including the variation of the roll angle.

An interesting feature of the preceding solution is illustrated in Fig. 2. Knowing the vehicle characteristics, L/D and W/C_LA, and the velocity and altitude at which the constant-altitude flight regime begins, the change in the heading angle is available immediately.

Reference

[1] Wang, H. E., "Motion of re-entry vehicles during constant-altitude glide," AIAA J. **3,** 1346–1348 (1965).

Reply by Author to R. T. Theobald

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In the course of our investigation of the lateral motion of re-entry vehicles at constant altitudes, assumptions other than $\sin\phi = 1$ have been examined. One assumption examined was precisely the one presented by Theobald in the preceding comment. Another one was a constant $\sin\phi$ different from unity, and a third one was a piecewise solution involving a series of values of $\sin\phi$. These approaches have not been pursued because they do not produce simple solutions for lateral range and downrange. It is true that the turn angle solution can be made extremely accurate, but it is very difficult sometimes to go beyond that to obtain useful expressions for lateral range and downrange.

The $\sin\phi = 1$ approximation used in Ref. 1 is quite a simplification to the physical behavior of the vehicle motion. Surprisingly enough, however, it is mathematically sufficiently accurate to describe the motion of the vehicle. Assuming $\sin\phi = 1$ introduces excessive lateral turning. On the other hand, the total distance traveled by the vehicle between a given initial velocity and the equilibrium glide velocity is not at all affected by this approximation. Consequently, the approximate solution (assuming $\sin\phi = 1$) demands only a redistribution of the distance traveled between lateral range and downrange. Because of the excessive lateral turning, the approximate solution always produces higher lateral range and lower downrange. The inaccuracy in lateral range and downrange, however, is much less than that in the turning angle.

The main values of Ref. 1 are that the solutions permit quick estimates of ranges for re-entry analysis, and they bring about the general trends of the various parameters involved. In this sense, a parallel can be drawn between the $\sin\phi = 1$ approximation in this problem and the straight-line trajectory approximation for ballistic re-entry.

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Comment on "Magnetohydrodynamic-Hypersonic Viscous and Inviscid Flow near the Stagnation Point of a Blunt Body"

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In a recent note,[1] Smith, Schwimmer, and Wu describe an extension to the magnetohydrodynamic, hypersonic, stagnation flow theory proposed by Bush.[2] In principle, their development involves including the viscous term in the constant property analysis and solving the two-point boundary value problem from both ends at once rather than starting at the shock and marching to the body. As a consequence of experimental and analytical studies in our laboratory, we wish to make several comments regarding this contribution.

Concerning the viscosity, the authors of Ref. 1 have assumed a thin viscous boundary layer, so that although included in the differential equations, viscosity is neglected in their shock vorticity jump boundary condition (from the curl of the momentum equation evaluated behind the shock using Rankins-Hugoniot conditions). This same assumption was made in earlier work[3] of the authors. The assumption is clearly valid if the shock layer Reynolds number is high. In the numerical examples of Ref. 1, $Re = v_\infty r_b/\nu = 100, 1000, \infty$. The corresponding shock layer Reynolds numbers are $Re_1 = \epsilon v_\infty r_b/\nu = 10, 100, \infty$ for $\epsilon = \frac{1}{10}$. Validity of the assumption at very low Re_1 requires detailed consideration.

First of all, from a qualitative viewpoint, it is necessary to recognize that when the Reynolds number is very low, the shock layer and the shock will be merged fully. The Hugoniot relations are then no longer valid and the full-fledged Navier-Stokes equations must be satisfied from the front of the shock to the body with the compressibility included in the shock region.

A more detailed evaluation concerns the extent of the boundary layer within the shock layer. We assume that the boundary-layer thickness δ is given by Hiemenz stagnation point flow.[4] Thus,

$$\delta \approx 2.4\,(\nu/a)^{1/2} \qquad (1)$$

where ν is kinematic viscosity and a is a constant determined by the flow outside of the boundary layer. If we match this flow to the Hugoniot shock downstream pressure, we obtain

$$\delta/\Delta \approx 2.4\, Re_\Delta^{-1/2} \qquad (2)$$

where Δ is the shock standoff distance and Re_Δ the shock layer Reynolds number based on Δ. If we denote by Re_1, the corresponding Reynolds number based on body radius r_b, we get

$$\delta/\Delta \approx 2.4[(\Delta/r_b)Re_1]^{-1/2} \qquad (3)$$

For a shock density ratio of $\epsilon = \frac{1}{10}$ and $Re_1 = 100$, it is shown in Ref. 1 that $\Delta/r_b \approx 0.1$. Hence

$$\delta/\Delta \approx 2.4/3.16 = 0.76$$

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It appears that the boundary layer occupies about $\frac{3}{4}$ of the shock layer, a sizable fraction. Although the foregoing calculations are very approximate, they do indicate that perhaps the Reynolds number term should be included in the forementioned boundary condition [Eq. (6) of Ref. 1]. This should present no fundamental difficulty if the quazilinearization technique is used because although the particular boundary condition would involve a linear combination of two functions, such a combination can be treated easily in the linear theory of differential equations.

In view of the forementioned we believe that if the Reynolds number were to be retained in the boundary conditions, the validity of the analysis would probably be extended down to $Re_1 \approx 100$. Secondly, we wish to call attention to the magnetic field boundary condition of Ref. 1. The body is located at the dimensionless variable $\eta = 1$ and the dimensionless magnetic field function is given in Ref. 1 by $g(\eta = 1) = g(1) = 1$ and $g'(1) = -1$. These are the boundary conditions needed to evaluate g which determines the magnetic field H. For example, in the radial direction,

$$H_r = H_0 \, [2g(\eta)/\eta^2] \cos\theta \tag{4}$$

where θ is the lateral angle. At $\theta = 0$, $\eta = 1$ we find $H_r(1) = 2H_0$ although presumably it should be $H_r(1) = H_0$. It appears as though Eq. 8 of Ref. 1 should read

$$g(1) = \tfrac{1}{2} \, (dg/d\eta)_{\eta=1} = -\tfrac{1}{2} \tag{5}$$

instead of +1 and −1, respectively.

The magnetic interaction parameter is

$$Q_m = (\mu_e H_0)^2 \sigma r_b / \rho_\infty v_\infty \tag{6}$$

where μ_e is the permeability, σ is the electric conductivity, and $\rho_\infty v_\infty$ are the freestream density and velocity, respectively. Thus, for any given value of Q_m, it appears that everywhere the magnetic field is twice as great as it should be. For this observation one of us (R. W. Porter) is indebted to W. Ericson and A. Maciulaitus.[5]

It appears that this apparent error can be corrected by replacing Q_m by $Q_m/4$, where Q_m is defined in Eq. (6). Thus, the graph of Ref. 1 would have an abscissa labeled $Q_m/4$ instead of Q_m. We note that this new $Q_m/4$ corresponds roughly to Bush's definition of the magnetic interaction parameter in terms of the dipole moment rather than the reference magnetic field. There is only a slight difference in that Bush used the shock radius r_s instead of the body radius r_s for the characteristic length and the point of evaluation of the reference magnetic field. Indeed, if one compares the curve attributed to Bush in Fig. 1 of Ref. 1 with Bush's tabulated values, one finds that the abscissa of this figure should be interpreted as $Q_m/4$.

Finally, we wish to comment on the large difference between the results of shock standoff at $Re = \infty$, $\epsilon = \frac{1}{10}$, and those of Bush for $Re = \infty$, $\epsilon = \frac{1}{11}$. Lighthill's result substantiates both results for zero magnetic interaction parameter[1,2] but the authors of Ref. 1 show a much larger increase in the standoff distance with nonzero magnetic parameter. This difference appears much larger than can be explained by the 10% difference in ϵ. The difference is not resolved by simply changing the abscissa of the forementioned figure, since all the curves are subject to the same change in scale. Ericson[5] has suggested that the difficulty may lie in the numerical calculations since the technique of Ref. 1 apparently fails to converge beyond a certain magnetic interaction parameter. We note that this limiting parameter does not correspond to the critical interaction parameter (evaluated at the shock) of Bush, now associated with shock layer liftoff, because the results of Bush[2] can be replotted vs the magnetic interaction parameter used in Ref. 1, and no such termination then occurs.

References

[1] Smith, M. C., Schwimmer, H. S., and Wu, C., "Magnetohydrodynamic-hypersonic viscous and inviscid flow near the stagnation point of a blunt body," AIAA J. 3, 1365–1367 (1965).

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[4] Schlicting, H., *Boundary Layer Theory*, transl. by J. Kestin (McGraw-Hill Book Co., Inc., New York, 1960), p. 81.

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Reply by Authors to R. W. Porter and A. B. Cambel

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TO acknowledge the three comments by Porter and Cambel,[1] we present the following reply. First of all, in our previous short note[2] the emphasis was put upon the demonstration of the computational scheme rather than on the physics of the problem. We made the assumption that the viscous effect right behind the shock wave is not important for computational convenience only. We made the same comment in an earlier work[3] that, for small Reynolds number, the thickness of shock layer and boundary layer may be of the same order of magnitude, and the usual Hugoniot conditions may be affected. Concerning the second comment, we remark that our definition of the dimensionless parameter Q_m is different from the parameter Q defined by Bush. It is true that in Fig. 1 of Ref. 2 Bush's result is incorrectly plotted. However, the comparison of our results with those obtained by Bush is not significant in any case. Lastly, we do not understand the third comment. The results of shock standoff distances for $\epsilon = \frac{1}{11}$, $Q_m = 0$ (obtained by Bush) and for $\epsilon = \frac{1}{10}$, $Q_m = 0$ (by us) do agree with that obtained by Lighthill. Furthermore, it is reasonable to expect that near the stagnation point, $\rho/\epsilon \approx$ const for fixed freestream velocity and density, and body radius. In our discussion, we have

$$\frac{\rho(\epsilon = \frac{1}{10}, Q_m = 0)}{\rho(\epsilon = \frac{1}{11}, Q_m = 0)} = \frac{0.073}{0.067} \approx \frac{11}{10}$$

The reasonableness of the results is therefore apparent.

References

[1] Porter, R. W. and Cambel, A. B., "Comment on 'Magnetohydrodynamic-hypersonic viscous and inviscid flow near the stagnation point of a blunt body,'" AIAA J. 4, 952–953.

[2] Smith, M. C., Schwimmer, H. S., and Wu, C.-S., "Magnetohydrodynamic-hypersonic viscous and inviscid flow near the stagnation point of a blunt body," AIAA J. 3, 1365–1367 (1965).

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